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Antigravity and classical solutions of five-dimensional Kaluza–Klein theory

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Abstract. We exhibit classical solutions of a graviton–graviphoton–graviscalar field theory which are antigravitating in the weak-field approximation. The theory itself is obtained by a Kaluza–Klein type reduction from five to four dimensions. The solutions are dyonic black holes with scalar charge. They share some similarities with the extreme Reissner–Nordstrom black holes of Einstein–Maxwell theory.

1. Introduction

This work was originally motivated by interest in investigating what Scherk (1980) calls 'antigravity'. In essence this phenomenon is one where the gravitational and other forces between certain objects in a field theory can mutually cancel in the Born approximation. This can occur when the structure of the part of the Lagrangian describing the coupling of objects in the theory to the dynamical fields that give rise to exchange forces imposes relationships between the coupling constants in the theory.

In this paper, we first review work by Scherk describing the sense in which classical solutions (loosely referred to as 'objects' above) of certain theories can be antigravitating in the weak-field limit. In particular we concentrate on his suggestion that Kaluza–Klein reduction of five-dimensional gravity to four dimensions provides a theory in which one could search for tensor–vector–scalar antigravitating objects.

The main work presented here (§§ 4 and 5) is a search amongst presently known classical solutions of the candidate theory for tensor–vector–scalar antigravitating objects. We show that one of these classical solutions is a plausible antigravitating object if we allow the vector exchange force to include magnetic as well as the usual electric interaction. This solution has properties somewhat similar to those of the extreme Reissner–Nordstrom solution of the Einstein–Maxwell system which is a tensor–vector antigravitating object.

2. Antigravity in classical solutions

In extended supersymmetries there are supermultiplets in which fields of lower spin occur as natural partners of the graviton. This admits the possibility that, in coupling such multiplets to others, relationships between the coupling strength of the graviton and of its lower-spin partners may arise. As Scherk (1980) has explained the simplest model in which this occurs is $N = 2$ supergravity. In $N = 2$ supergravity, the graviton's multiplet contains $e^r_\mu(x)$, the vierbein; $\psi^i_\mu(x)$, two Majorana gravitini, and $A_\mu(x)$, a vector field. $A_\mu(x)$ gauges the central charge Z of $N=2$ supersymmetry and couples to massive matter multiplets by appearing in covariant derivatives

$$\mathscr{D}_\mu = \partial_\mu - \mathrm{i}gA_\mu$$

where $g=\kappa M$, M is the mass of the massive multiplet and $\kappa^2 = 4\pi G$, with G as Newton's constant.

The important point is that since Z has the dimensions of mass, A_μ has a mass-dependent coupling (for this reason it is often referred to as a graviphoton), as does the graviton. Thus in the static, spherically symmetric, weak-field approximation to the force between two massive particles, the potential is found to be (Zachos 1978)

$$V(r) = \frac{\kappa^2}{4\pi r}\left(MM' - \frac{gg'}{\kappa^2}\right) = -\frac{\kappa^2}{4\pi r}MM'(1-\varepsilon\varepsilon')$$

where we have used

$$g = \varepsilon\kappa M$$

with

$$\varepsilon = \begin{cases} +1 & \text{particle} \\ -1 & \text{antiparticle.} \end{cases}$$

Evidently the force between a widely separated particle (or antiparticle) pair vanishes, though it is doubled between a particle–antiparticle pair. The case of vanishing force due to graviton–graviphoton exchange cancellation is an example of what Scherk calls antigravity.

The $N=2$ Lagrangian density for the $\{e^r_\mu, \psi^i_\mu, A_\mu\}$ multiplet reduces to

$$\mathscr{L}_{\mathrm{red}} = \frac{e}{4\kappa^2}R - \frac{e}{4}F_{\mu\nu}F^{\mu\nu}$$

if we set $\psi^i_\mu = 0$ as we should if we are concerned with classical tensor–vector effects. Looking for an antigravitating object necessitates solving the field equations arising from $\mathscr{L}_{\mathrm{red}}$. Such an object would need to be static and spherically symmetric. Its charge and mass would need to be related by $Q^2 = \kappa^2 M^2$.

Now $\mathscr{L}_{\mathrm{red}}$ is just the Einstein–Maxwell Lagrangian and the well known Reissner–Nordstrom solutions have most of the properties we desire. Choosing $\kappa^2 = 1$ the solutions are

$$\begin{aligned} \mathrm{d}s^2 &= -B(r)\,\mathrm{d}t^2 + \mathrm{d}r^2/B(r) + r^2\,\mathrm{d}\Omega \\ A_\mu &= \{A_t, 0, 0, 0\} \qquad A_t = Q/4\pi r \end{aligned} \tag{2.1}$$

where

$$B(r) = 1 - \frac{2m}{4\pi r} + \frac{Q^2}{(4\pi)^2 r^2}.$$

These objects are electrically charged black holes if $M^2 \geq Q^2$. The tensor–vector antigravity condition $Q^2 = M^2$ singles out the extreme black hole. These black holes are antigravitating in the sense that two such objects if widely separated would experience no net force due to the exchange of virtual gravitons and photons in the case where forces were governed by the $N=2$ supergravity Lagrangian.

3. Kaluza–Klein reduction

Kaluza–Klein type reduction—from five to four dimensions—of the Einstein Lagrangian provides a Lagrangian for the graviton, graviphoton and graviscalar. These fields all couple to external fields in a mass-dependent way and again there is the possibility of weak-field force cancellation.

In particular, consider the five-dimensional Lagrangian density ($K^2 = 1$)

$$\begin{aligned}\mathcal{L}_5 &= \mathcal{L}_5^{\mathrm{E}} + \mathcal{L}_5^{\mathrm{m}} \\ &= \tfrac{1}{4}\hat{e}\hat{R} + \hat{e}g^{\hat{\mu}\hat{\nu}}\partial_{\hat{\mu}}\phi^*\partial_{\hat{\nu}}\phi.\end{aligned} \tag{3.1}$$

(The symbol ^ denotes five dimensions, and the superscripts E and m are for 'Einstein' and matter.) We assume that the five-dimensional space–time has a Killing vector associated with a compact dimension with S^1 (circular) topology. Taking our four-dimensional world to be the hypersurface orthogonal to the Killing vector, we can introduce the following *ansatz* for the space–time dependence of our fields:

$$\begin{aligned}\hat{e}^{\hat{r}}_{\hat{\mu}}(x, y) &= \hat{e}^{\hat{r}}_{\hat{\mu}}(x) \\ \varphi(x, y) &= \varphi(x)\exp(\mathrm{i}my)\end{aligned} \tag{3.2}$$

and we may parametrise the funfbein by (Scherk 1980)

$$\hat{e}^{\hat{r}}_{\hat{\mu}}(x) = \begin{array}{c} \\ x^1 \\ \vdots \\ x^4 \\ \\ y \end{array}\begin{array}{c} \begin{array}{cc} x^1 \ldots x^4 & \qquad\qquad y \end{array} \\ \left[\begin{array}{c|c} \exp\left(\dfrac{-\sigma(x)}{\sqrt{3}}\right)e^{r}_{\mu}(x) & 2A_\mu(x)\exp\left(\dfrac{2\sigma(x)}{\sqrt{3}}\right) \\ \hline & \exp\left(\dfrac{2\sigma(x)}{\sqrt{3}}\right) \end{array}\right]\end{array}. \tag{3.3}$$

$A_\mu(x)$ and $\sigma(x)$ will be our graviphoton and graviscalar, respectively. With this choice

$$\mathcal{L}_5^{\mathrm{E}} \to \mathcal{L}_4 = -\tfrac{1}{4}eR + \tfrac{1}{4}e\exp(2\sqrt{3}\sigma)F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma. \tag{3.4}$$

The fifth dimension now becomes a gauged internal dimension as can be verified locally by noting that, with our *ansatz*, coordinate transformations in the fifth dimension reduce to

$$\begin{aligned}&\delta A_\mu(x) = \tfrac{1}{2}\partial_\mu\xi^5_{(x)} \qquad\qquad \delta\varphi(x) = \mathrm{i}m\xi^5_{(x)}\varphi(x) \\ &\delta\sigma(x) = 0 = \delta e^{r}_{\mu}(x).\end{aligned} \tag{3.5}$$

$$\mathcal{L}_5^{\mathrm{m}} \to \mathcal{L}_4^{\mathrm{m}} = e[g^{\mu\nu}(\partial_\mu + \mathrm{i}2mA_\mu)\varphi^*(\partial_\nu - \mathrm{i}2mA_\nu)\varphi - m^2\varphi^*\phi\exp(-2\sqrt{3}\sigma)]. \tag{3.6}$$

In (3.6) we see the mass-dependent coupling of the graviphoton and graviscalar.

The weak-field static potential is

$$V(r) = -\frac{1}{4\pi r}mm'(1 - 4\varepsilon\varepsilon' + 3). \tag{3.7}$$

Thus, the force between two identical objects has (apart from the gravitational attraction), an 'electromagnetic' repulsion with charge Q where $Q^2 = 4M^2$, and a scalar attraction with an effective scalar charge s where $s^2 = 3M^2$. (By analogy with the electric case, we let the scalar charge of an object be the coefficient of $1/4\pi r$ in the asymptotic expansion of its scalar field). This condition

$$Q^2 = 4M^2 \qquad s^2 = 3M^2 \tag{3.8}$$

will be called the *Scherk antigravity* condition. Objects whose mass, electric charge and scalar charge are so related, will experience no mutual force when widely separated.

At this stage it is important to notice that the parametrisation chosen in equation (3.3) is equivalent to the following parametrisation of the five-dimensional metric:

$$\hat{g}_{\hat{a}\hat{b}} = \begin{pmatrix} (1/\omega)g^{(4)}_{\mu\nu} + 4A_\mu A_\nu \omega^2 & 2A_\mu \omega^2 \\ 2A_\nu \omega^2 & \omega^2 \end{pmatrix} \tag{3.9}$$

where $\omega^2(x) = \exp(4\sigma(x)/\sqrt{3})$.

This differs from the more usual parametrisation where the four-dimensional metric $\gamma^{(4)}_{\mu\nu}$ is chosen so that

$$\gamma^{(4)}_{\mu\nu} = g^{(4)}_{\mu\nu}/\omega. \tag{3.10}$$

It is important to take this conformal relationship into account when identifying the various charges in classical solutions.

In the next two sections we will investigate what happens when we impose the relationships (3.8) on the known static, spherically symmetric solutions of the field equations derived from (3.4).

4. The CD solution

Several authors (Chodos and Detweiler 1981, Dobiasch and Maison 1981, Belinski and Ruffini 1980) have presented classical solutions of the field equations derived from field theories equivalent to (3.4). In this section we look for antigravitating objects in the solutions due to Chodos and Detweiler (CD) (Note that the solutions of Belinski and Ruffini may immediately be omitted from consideration because they are rotating, i.e. not static.)

Written in terms of the quantities defined in (3.9) the CD solution is (space–time coordinates t, r, θ, φ):

$$\omega = \psi^{1/2}[a_1\psi^{(1+K)^{1/2}} + (1-a_1)\psi^{-(1+K)^{1/2}}]^{1/2}$$

$$A_\mu = \{A_t, 0, 0, 0\}$$

with

$$A_t = \tfrac{1}{2}[a_1(a_1-1)]^{1/2} \frac{[\psi^{(1+K)^{1/2}} - \psi^{-(1+K)^{1/2}}]}{[a_1\psi^{(1+K)^{1/2}} + (1-a_1)\psi^{-(1+K)^{1/2}}]} \tag{4.1}$$

$$ds^2 = g^{(4)}_{\mu\nu}\, dx^\mu\, dx^\nu = -\frac{(r^2-B^2)^2}{r^2 r_s^2}\, dt^2 + \frac{r_s^2}{r^2}\, dr^2 + r_s^2\, d\Omega$$

where

$$\psi = \left(\frac{r-B}{r+B}\right)^{\lambda/2B} \qquad \lambda = \frac{4B}{(K+4)^{1/2}}$$

$$r_s = \frac{(r^2-B^2)}{r\psi^{3/4}}[a_1\psi^{(1+K)^{1/2}} + (1-a_1)\psi^{-(1+K)^{1/2}}]^{1/4}$$

a_1, K and B are the three (constant) parameters that characterise a solution.

The field equations derived from (3.4) show that the curvature scalar (R) is given by

$$R \propto g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma,$$

hence

$$R \propto \frac{\lambda^2 r^4 \psi^{3/2}}{(r^2-B^2)^4} \frac{\{a_1[1+(1+K)^{1/2}]\psi^{(1+K)^{1/2}}+(1-a_1)[1-(1+K)^{1/2}]\psi^{-(1+K)^{1/2}}\}^2}{[a_1\psi^{(1+K)^{1/2}}+(1-a_1)\psi^{-(1-K)^{1/2}}]^{5/2}}. \tag{4.2}$$

We identify the scalar charge (s), gravitational mass (M) and electric charge (Q) by the following asymptotic formulae:

$$\begin{aligned} &\omega \sim 1+\frac{2}{\sqrt{3}}\frac{s}{4\pi r} \\ &g_{00} \sim 1-2M/4\pi r \\ &A_t \sim Q/4\pi r \qquad \text{as } r\to\infty. \end{aligned} \tag{4.3}$$

Note that $r \to r_s$ as $r\to\infty$ where r_s is a 'true' radial coordinate in the sense that $4\pi r_s^2$ is the area of a sphere centred at the origin. Thus, the asymptotic expansion in (4.3) in terms of r is the same as would be obtained when expanding in terms of r_s.

Using (4.3) we obtain the following expressions for the charges in terms of the parameters of the solutions:

$$\begin{aligned} &s = -\pi\sqrt{3}\lambda[1+(2a_1-1)(1+K)^{1/2}] \\ &M = 4\pi\lambda - \pi\lambda[1+(2a_1-1)(1+K)^{1/2}] \\ &Q = -4\pi[a_1(a_1-1)\lambda^2(1+K)]^{1/2}. \end{aligned} \tag{4.4}$$

From these expressions in turn we obtain

$$\begin{aligned} &4\pi\lambda = M - s/\sqrt{3} \\ &a_1 = \tfrac{1}{2}\left(1-\frac{M\sqrt{3}+3s}{(M\sqrt{3}-s)(1+K)^{1/2}}\right) \\ &B = \frac{1}{8\pi}(M^2+s^2-Q^2)^{1/2}. \end{aligned} \tag{4.5}$$

An obvious generalisation of equation (3.7) shows that the force between two widely separated objects with mass, electric and scalar charge is

$$\text{Force} \propto (QQ'-MM'-ss')/r^2. \tag{4.6}$$

From equations (4.5) and (4.6) we see that there is a whole class of CD metrics which will experience zero force between widely separated identical pairs, i.e. antigravity. These are the metrics with $B=0$. (The Scherk antigravity condition, $s^2=3M^2$, $Q^2=4M^2$, will be one of these metrics).

In the limit $B=0$

$$K=-4 \qquad \text{and} \qquad \psi\to\exp(-\lambda/r).$$

Now the Scherk condition requires

$$s=\pm\sqrt{3}\,M,$$

but note that

$$4\pi\lambda = M - s/\sqrt{3}.$$

So $s = +\sqrt{3}\, M$ would imply $\lambda = 0$ and indeed the solution would be the trivial, flat, empty-space one. We therefore implement Scherk's condition by letting

$$s = -\sqrt{3}\, M.$$

Then

$$\lambda = M/2\pi \qquad \text{and} \qquad a_1 = \tfrac{1}{2} - \mathrm{i}/2\sqrt{3}.$$

From (4.2) we easily see that for these metrics there is a physical singularity when

$$[\cos(\sqrt{3}\lambda/r) - (1/\sqrt{3})\sin(\sqrt{3}\lambda/r)] = 0,$$

i.e. at

$$r_0 = 2\sqrt{3}\, M/\tan^{-1}(\sqrt{3}).$$

This singularity is not clothed by any horizon and the space–time therefore has a naked singularity, as do all other ($B = 0$) antigravitating CD metrics.

Thus insofar as we believe naked singularities to be non-physical objects, we must conclude that the CD solutions do not provide a physical realisation of the antigravity condition.

5. The DM solution

The DM solutions of interest (excluding those with logarithmic and branch-type singularities) are (Dobiasch and Maison 1981)

$$\begin{aligned} &\omega^2 = B/A \\ &E_\mathrm{r} = -\frac{\delta}{2}\frac{A}{B^2} \qquad B_\mathrm{r} = -\frac{\gamma}{2}\frac{1}{(AB)^{1/2}} \\ &\mathrm{d}s^2 = g^{(4)}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -\frac{f^2}{(AB)^{1/2}}\mathrm{d}t^2 + \frac{(AB)^{1/2}}{f^2}\mathrm{d}\tilde{r}^2 + (AB)^{1/2}\,\mathrm{d}\Omega \end{aligned} \tag{5.1}$$

where

$$\begin{aligned} &A(\tilde{r}) = \tilde{r}^2 - \frac{(\alpha^2 - a^2)(\alpha+\beta)}{4(\alpha-\beta)} \\ &B(\tilde{r}) = \left(\tilde{r} + \frac{\alpha+\beta}{2}\right)^2 - \frac{(\beta^2 - a^2)(\alpha+\beta)}{4(\beta-\alpha)} \\ &f^2(\tilde{r}) = \left(\tilde{r} + \frac{\alpha}{2}\right)^2 - \frac{a^2}{4} \\ &\gamma^2 = \frac{\alpha(a^2 - \alpha^2)}{(\beta-\alpha)} \qquad \delta^2 = \frac{\beta(\beta^2 - a^2)}{(\beta-\alpha)} \end{aligned} \tag{5.2}$$

α, β and a are the three (constant) parameters that specify a solution. E_r and B_r are the radial elecctric and magnetic fields respectively. We have used the notation $\tilde{r}$ for the DM radial coordinate which differs from the radial coordinate used by Chodos and Detweiler (1981).

The vector field for this solution is

$$A_t = \delta \tilde{r}/2B \qquad A_r = A_\theta = 0 \qquad A_\varphi = \tfrac{1}{2}\cos\theta. \tag{5.3}$$

In fact

$$A^\varphi = g^{\varphi\varphi} A_\varphi$$

and

$$A^\varphi \sim \frac{\gamma}{2\tilde{r}} \frac{\cos\theta}{\sin\theta} \qquad \text{as } \tilde{r} \to \infty.$$

Since $\tilde{r} \to r_s$ at infinity we see that these objects behave magnetically like Dirac monopoles (Olive and Goddard 1978). Since these objects are also electrically charged, they are dyons. Asymptotic expansions lead to the following expressions for the charges of the solution in terms of the parameters:

$$\begin{aligned} s &= \pi\sqrt{3}(\beta + \alpha) \qquad & M &= \pi(\beta - \alpha) \\ Q &= 2\pi\delta \qquad & P &= 2\pi\gamma \end{aligned} \tag{5.4}$$

where P is the magnetic charge. A little works yields the curvature scalar

$$R \propto \frac{f^2}{(AB)^{1/2}} \frac{(AB' - BA')^2}{B^2 A^2}. \tag{5.5}$$

(The prime denotes differentiation with respect to $\tilde{r}$.)

The immediate question is: does the DM solution look like the CD solution when the magnetic field is identically zero? Zero magnetic field means

$$\gamma^2 = \frac{\alpha(\alpha^2 - a^2)}{(\alpha - \beta)} = 0,$$

i.e.

$$\alpha = 0 \qquad \text{or} \qquad \alpha = a.$$

The comparison is most easily performed by conformally transforming both DM and CD metrics to (see (3.10))

$$ds^2 = \gamma^{(4)}_{\mu\nu}\, dx^\mu\, dx^\nu$$

and then comparing them. The question of equivalence is then

$$-e^\nu\, dt^2 + e^\beta\, dr^2 + e^\beta r^2\, d\Omega = -(f^2/B)\, dt^2 + (A/f^2)\, d\tilde{r}^2 + A\, d\Omega \tag{5.6}$$

when $\alpha = 0$ or $\alpha = a$ where

$$e^{\nu(r)} = \psi/[a_1 \psi^{(1+K)^{1/2}} + (1 - a_1)\psi^{-(1+K)^{1/2}}]$$

and

$$e^{\beta(r)} = (r^2 - B^2)^2 / r^4 \psi^2.$$

Evidently the t, θ and φ coordinates are the same in both metrics, but the radial coordinates r and $\tilde{r}$ may be different.

(i) When $\alpha = 0$:

$$A(\tilde{r}) = f^2(\tilde{r}).$$

If (5.6) is to be true, the relationship between radial coordinates is given by $f^2(\tilde{r}) = e^{\beta(r)}r^2$. Then the equivalence of the γ_{00} and scalar fields implies

$$e^{\nu} = f^2/B = A/B = 1/\omega_{\rm DM}^2 = 1/\omega_{\rm CD}^2,$$

i.e.

$$\psi^2(r)/\omega_{\rm CD}^2(r) = 1/\omega_{\rm CD}^2(r).$$

This inconsistency is only removed when the solutions are both trivially flat. So the $\alpha = 0$ limit of the DM solution does not yield the CD solutions.

(ii) When $\alpha = a$:

$$A = \tilde{r}^2 \qquad f^2 = \tilde{r}(\tilde{r} + \alpha)$$

$$B = \tilde{r}(\tilde{r} + \alpha + \beta).$$

The relationship between radial coordinates is

$$\tilde{r}^2 = e^{\beta(r)}r^2.$$

Equality of the γ_{00} would then imply

$$e^{\nu} = f^2/B$$

i.e.

$$\frac{\psi^2}{\omega_{\rm CD}^2} = \frac{e^{\beta/2}r + \alpha}{e^{\beta/2}r + \alpha + \beta}$$

which is again an inconsistency for the general CD solution.

Our conclusion is then that despite their inclusion of an extra magnetic field, the DM solutions are not more general than the CD ones, but merely different. We must now examine the antigravity metrics in these DM solutions.

Dobiasch and Maison (1981) have presented a concise analysis of the singularity structure of their solutions. Their results remain largely true after the conformal rescaling (3.10) because the canonical curvature scalar is again only singular when $A(r)$ or $B(r)$ vanishes (see (5.5)). However, the allowed ranges of the parameters α, β and a, compatible with the reality of γ and δ and positivity of the gravitational mass are now different because we are using M as given in (5.4) and so require $\beta > \alpha$. Choosing $a \geqslant 0$ (without losing anything), black hole space–times occur when $\beta \geqslant a$, $\alpha \leqslant -a$. The singularities are always pointlike and they are shielded from $r = +\infty$ by two horizons at $\tilde{r}_{\pm} = -\frac{1}{2}\alpha \pm \frac{1}{2}a$ (the solutions of $f^2 = 0$ which yield timelike coordinate singularities). The scalar charge of these black holes is not equal to zero and neither the scalar field nor the electromagnetic field is singular at either horizon. This contrasts with the behaviour of the black holes with scalar charge found by Bekenstein (1976).

Examination of the antigravity limit for these magnetically charged objects requires a knowledge of the weak force between two widely separated dyons. Olive and Goddard (1978) have shown how the guiding principle of dual symmetry can lead to a natural generalisation of the Lorentz force law that encompasses magnetically charged objects

$$m\, \mathrm{d}^2x^{\mu}/\mathrm{d}\tau^2 = (QF^{\mu\nu} + P{}^*F^{\mu\nu})\, \mathrm{d}x^{\nu}/\mathrm{d}\tau \tag{5.7}$$

where ${}^*F^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the dual of $F^{\mu\nu}$. Now the dual can be obtained by letting

$$\boldsymbol{E} \rightarrow \boldsymbol{B} \qquad \boldsymbol{B} \rightarrow -\boldsymbol{E}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field three-vectors respectively. Thus, (5.7) implies that magnetic charge couples in a way similar to electric charge. The force law (4.6) is immediately generalised to

$$\text{Force} \propto (QQ' + PP' - MM' - ss')\tilde{r}^2. \tag{5.8}$$

Thus an antigravitating dyon would be one for which

$$M^2 + s^2 = P^2 + Q^2. \tag{5.9}$$

Using (5.4) this becomes

$$a^2(\beta - \alpha) = 0.$$

Now $\beta = \alpha$ implies $M = 0$ which Witten (1981) has argued singles out flat Minkowski space uniquely. Thus the correct antigravity condition for the DM solution is $a = 0$. The Scherk antigravity metric will be one of these. In this limit the DM solutions are black holes if $\beta > 0$ and $\alpha < 0$. In general they have non-zero scalar, electric and magnetic charge. All these black holes have coincident $\tilde{r}_+ = \tilde{r}_-$ horizons and their singularities are pointlike. These then are extreme black holes and they satisfy tensor–vector–scalar antigravity in a way reminiscent of that in which extreme Reissner–Nordstrom holes satisfy tensor–vector antigravity.

Another interesting similarity emerges if we use standard black hole thermodynamics (Hawking 1975, Davies 1976) to evaluate the temperature of DM black holes:

$$T = \frac{4\pi^2 a M^2 (3M^2 - s^2)^{1/2}}{[as^2(M\sqrt{3} + s + \frac{1}{2}a\sqrt{3})(M\sqrt{3} - s + \frac{1}{2}a\sqrt{3}) + 3M(2M + a)(M + 8\pi^2 a)(3M^2 - s^2)]} \tag{5.10}$$

where $a = (M^2 + s^2 - P^2 - Q^2)^{1/2}/2\pi$ is the normal DM parameter, and we have chosen Boltzmann's constant $k = 48\pi\sqrt{3}$. Thus the antigravity DM black holes (with $a = 0$) have a black hole temperature of zero which is again a property of extreme Reissner–Nordstrom black holes.

6. Discussion

We have found in the extreme DM black holes, plausible weak-field tensor–vector–scalar antigravitating objects. The space–time part of this object is a black hole with (in general) non-zero scalar, electric and magnetic charges and non-zero gravitational mass. It mimics some of the important properties of the extreme Riessner–Nordstrom metric which is tensor–vector antigravitating. Hajicek (1981) has shown that the extreme Reissner–Nordstrom black hole (with magnetic charge only) is also the only gravitational soliton in Einstein–Mawell theory. We could speculate that it would be interesting to establish whether the DM extreme black hole could be a new member of the set of gravitational solitons. (This question of new gravitational solitons is also being pursued in the context of $N = 2$ supergravity by Gibbons (1981a, b).)

Even from a classical standpoint it would be interesting to check the stability of the scalar charge of these black holes. It would then be clear whether we need to modify present 'no-hair theorems' and let scalar charge join the list of stable black hole parameters.

A check to see whether the antigravitating behaviour extends beyond the weak-field limit is another interesting problem.

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